IKIR FEB RAS

Application of adaptive wavelet thresholding to recovery geophysical signal pulse waveforms

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Problem

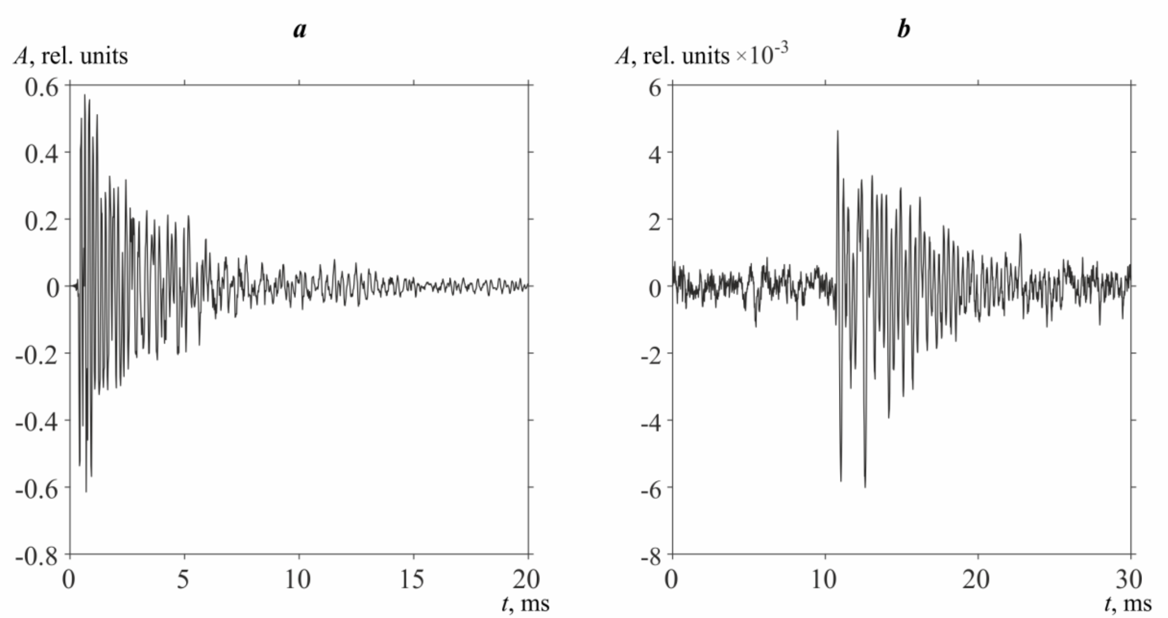


Figure 1. Geoacoustic pulses

Reasons

- natural noise
- artificial interference
- nonlinearity of receive path
- dynamic range limitations
- quantization errors
- primary hardware processing, etc

Estimation of background noise parameters

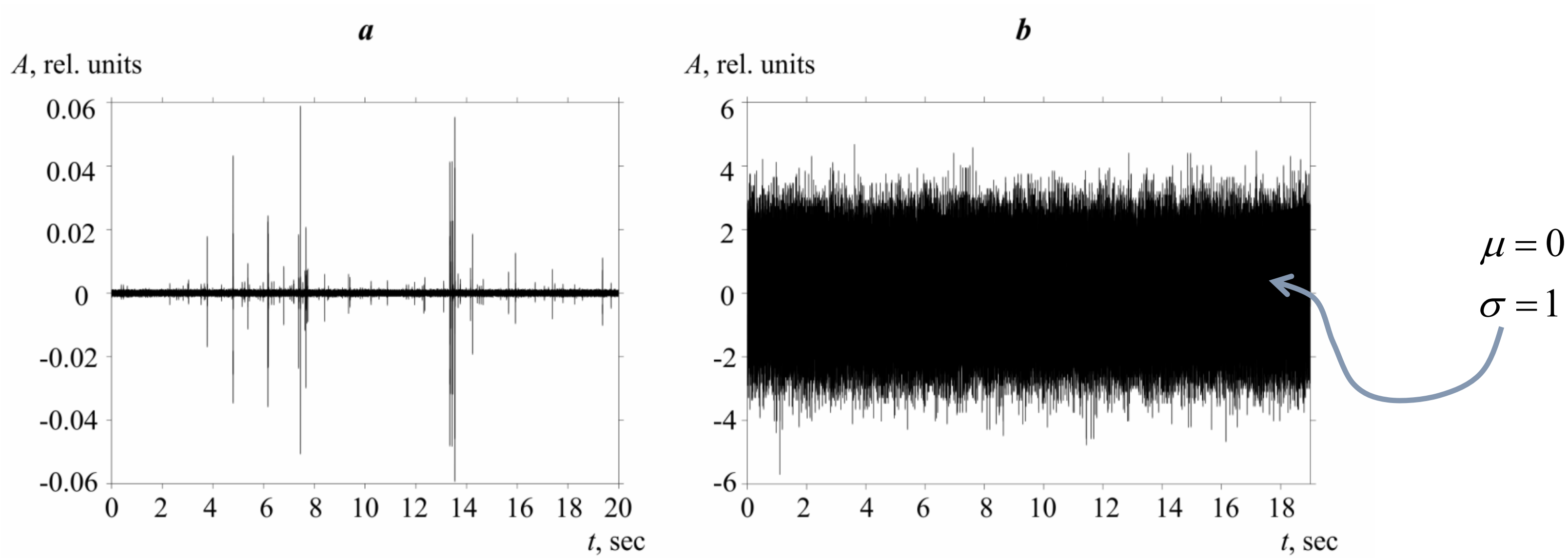


Figure 2. ***a*** – pulsed signal; ***b*** – noise

Fragments without pulses were selected from the geoacoustic signal recorded in good weather conditions. This noise signal was scaled and shifted.

Estimation of background noise parameters

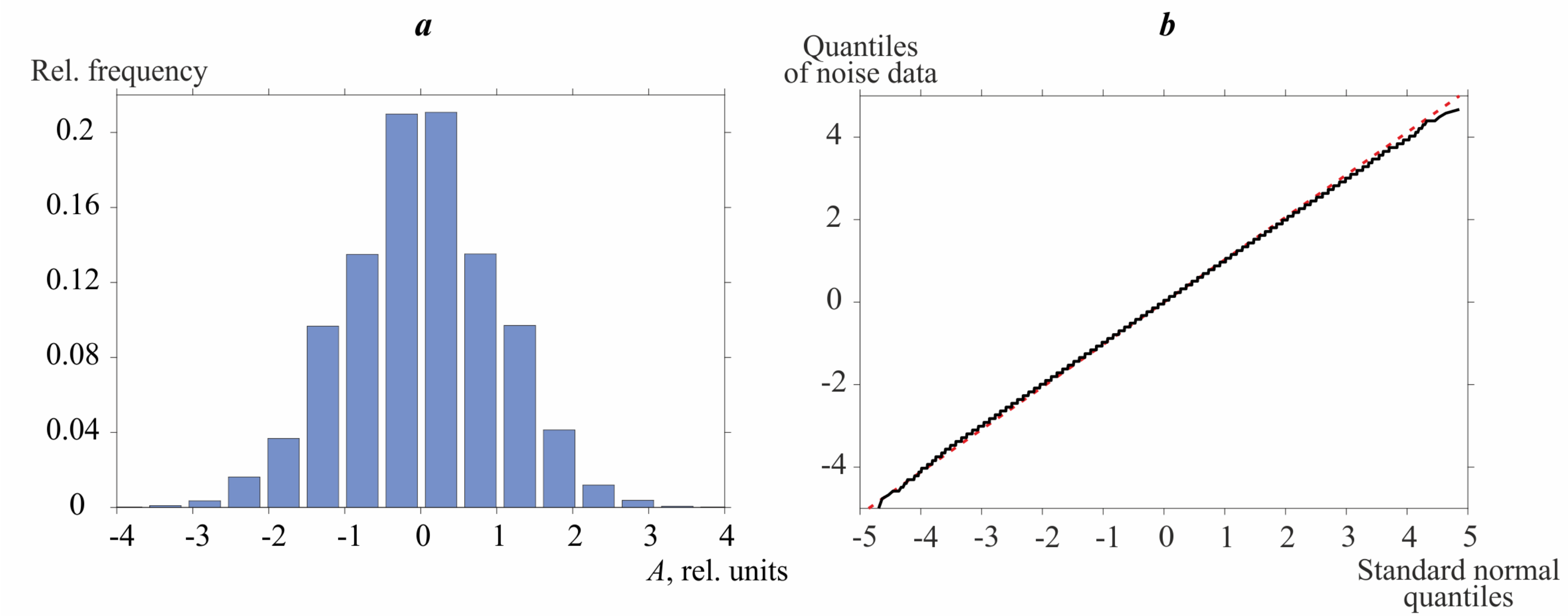


Figure 3. ***a*** – histogram; ***b*** – Q-Q plot

Estimation of background noise parameters

Table 1. Applying various statistical test

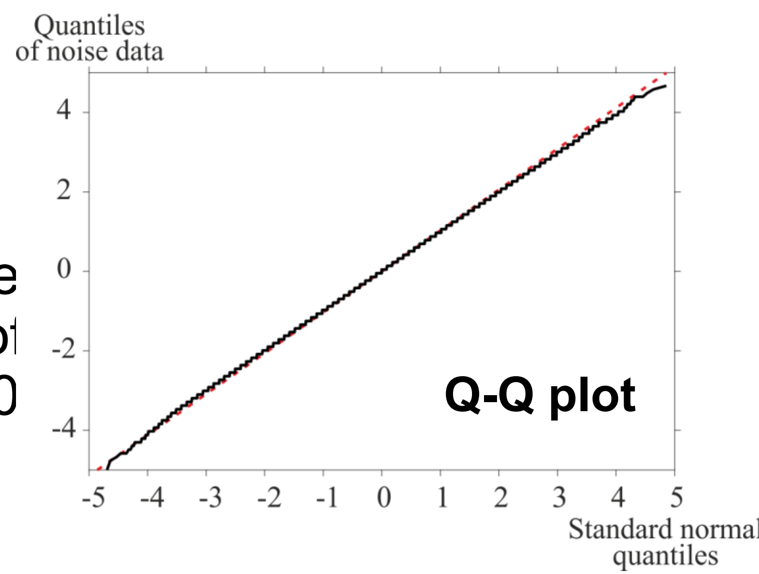
Test	Significance level, α	Sample size, N	Accepted hypothesis, H_0 or H_1
Pearson	0.05	500	H_0
		1000	H_0
Anderson–Darling	0.05	500	H_0
		1000	H_1
Lilliefors	0.05	500	H_0
		1000	H_1

H_0 – The noise signal amplitudes have the normal distribution $N(\mu, \sigma)$, where μ and σ are estimated from the tested data

Estimation of background noise parameters

- ADC used in the signal registration system has a resolution of 16 bits (65536 levels, values from −1 to 1)
- According to calculations, dynamic range of the formed noise signal contains 112 levels (values from −0.00189 to 0.001526)

- Gaussian noise signal was generated and digitized
- The Wilcoxon–Mann–Whitney te confirmed H_0 (the distributions of both samples are equal) at α = 0 for N = 500, 1000



Wavelet thresholding as denoising method

Forward Wavelet Transform → Thresholding → Inverse Wavelet Transform

- Wavelet family
  - Daubechies wavelets
  - symlets
  - coiflets

- Strategy for choosing a threshold value → adaptive threshold
  - Universal threshold
  - Minimax
  - FDR
  - SURE
  - Block James-Stein
  - Empirical Bayes method

- Thresholding scheme
  - hard
  - soft
  - special schemes for certain methods

Method selection

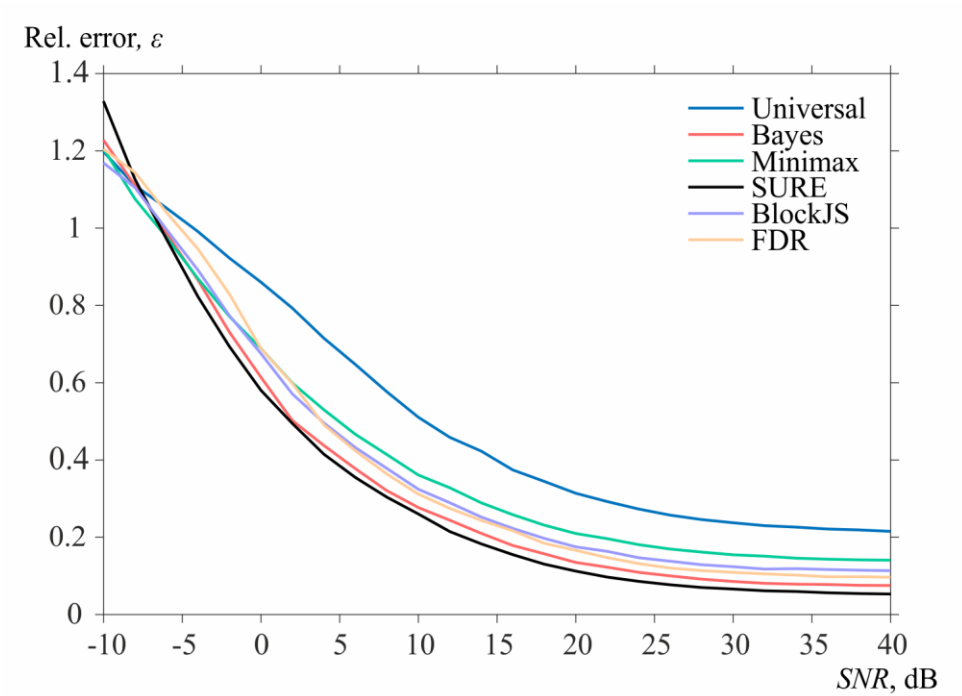


Figure 4. ε(*SNR*) plots for different threshold calculation methods

Average relative error of wavelet thresholding

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\frac{\|s_i(t)-\hat{s}_i(t)\|}{\|s_i(t)\|} \tag{1}$$

where

$s_i(t)$ is an undistorted pulse

$\hat{s}_i(t)$ is a pulse after the wavelet thresholding

N is a total number of processed pulses

Empirical Bayes Method

Mathematics

$$x_i = \mu_i + \epsilon_i \qquad (2)$$

where

x_i are detail wavelet coefficients of distorted signal

μ_i are coefficients of undistorted signal

ϵ_i is normally distributed noise

$$f(\mu) = (1-\omega)\delta_0 + \omega\gamma(\mu) \qquad (3)$$

where

ω is the probability that $\mu_i = 0$

γ is quasi-Cauchy distribution

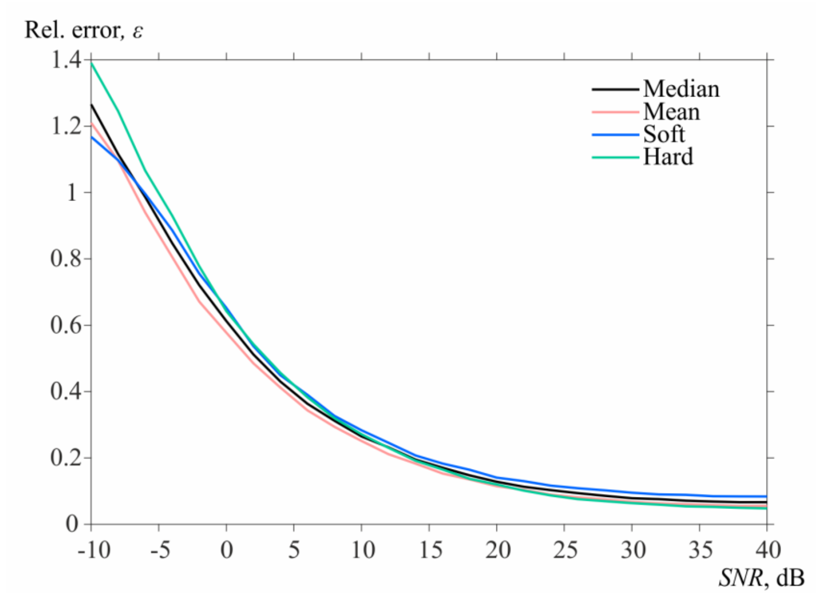


Figure 5. $\varepsilon(SNR)$ plots for various thresholding procedures

Thresholding procedure

- μ_i are evaluated by posterior median $\hat{\mu}(d_i, \omega)$
- μ_i are evaluated by posterior mean $\bar{\mu}(d_i, \omega)$
- determining $t(\omega)$ and soft or hard thresholding

Wavelet family selection

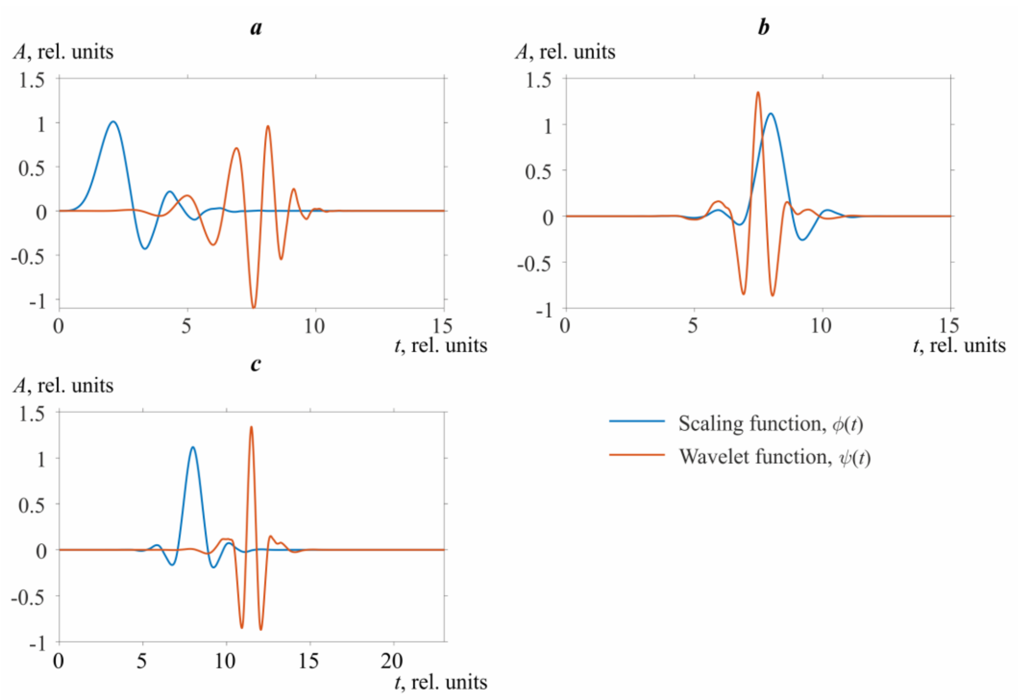


Figure 6. Mother wavelets and scaling functions:

a — eighth-order Daubechies wavelet, db_8;

b — eighth-order symlet, sym_8;

c — forth-order coiflet, $coif_4$

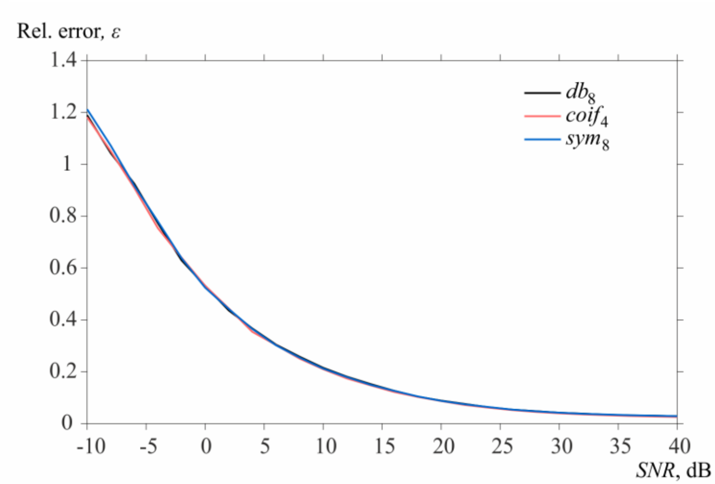


Figure 7. $\varepsilon(SNR)$ plots obtained by the Empirical Bayes Method using various wavelet families

Approbation

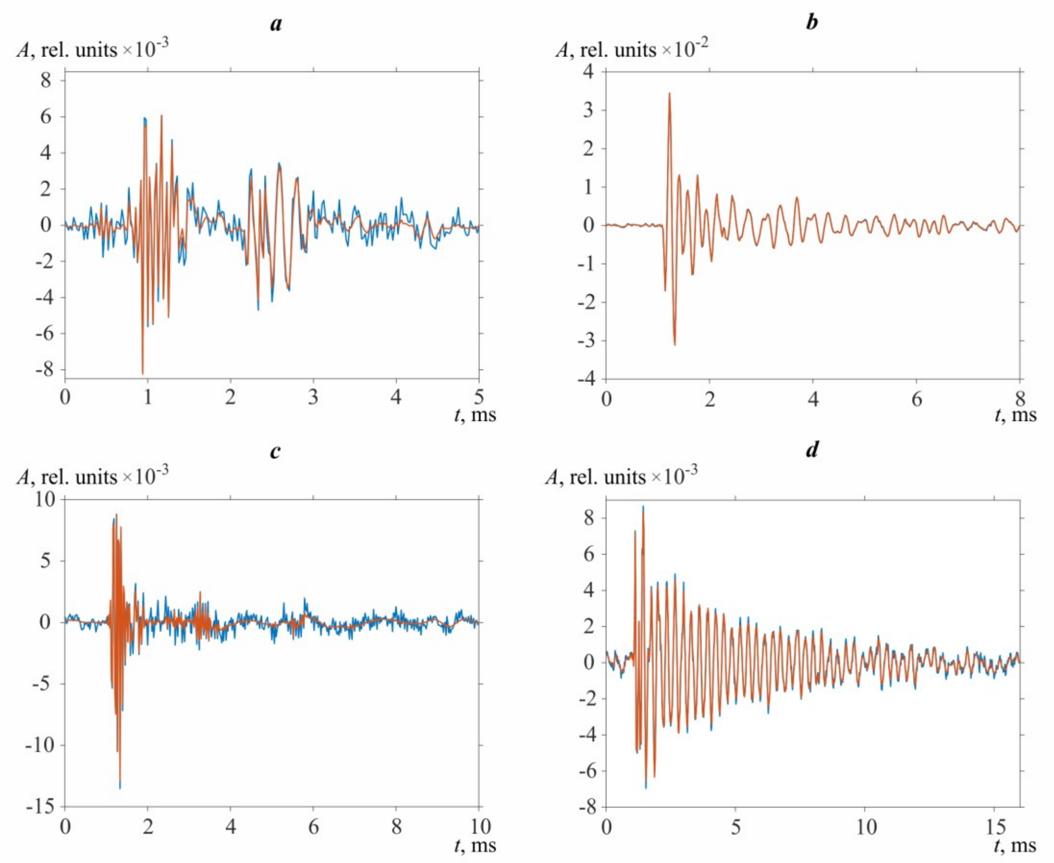


Figure 8. Real data processing

Denoising method

- Empirical Bayes Method
- posterior mean
- forth-order coiflets

Thank you
for your attention

Questions