Nonextensive analysis of natural and technogenic seismicity of Sakhalin Island

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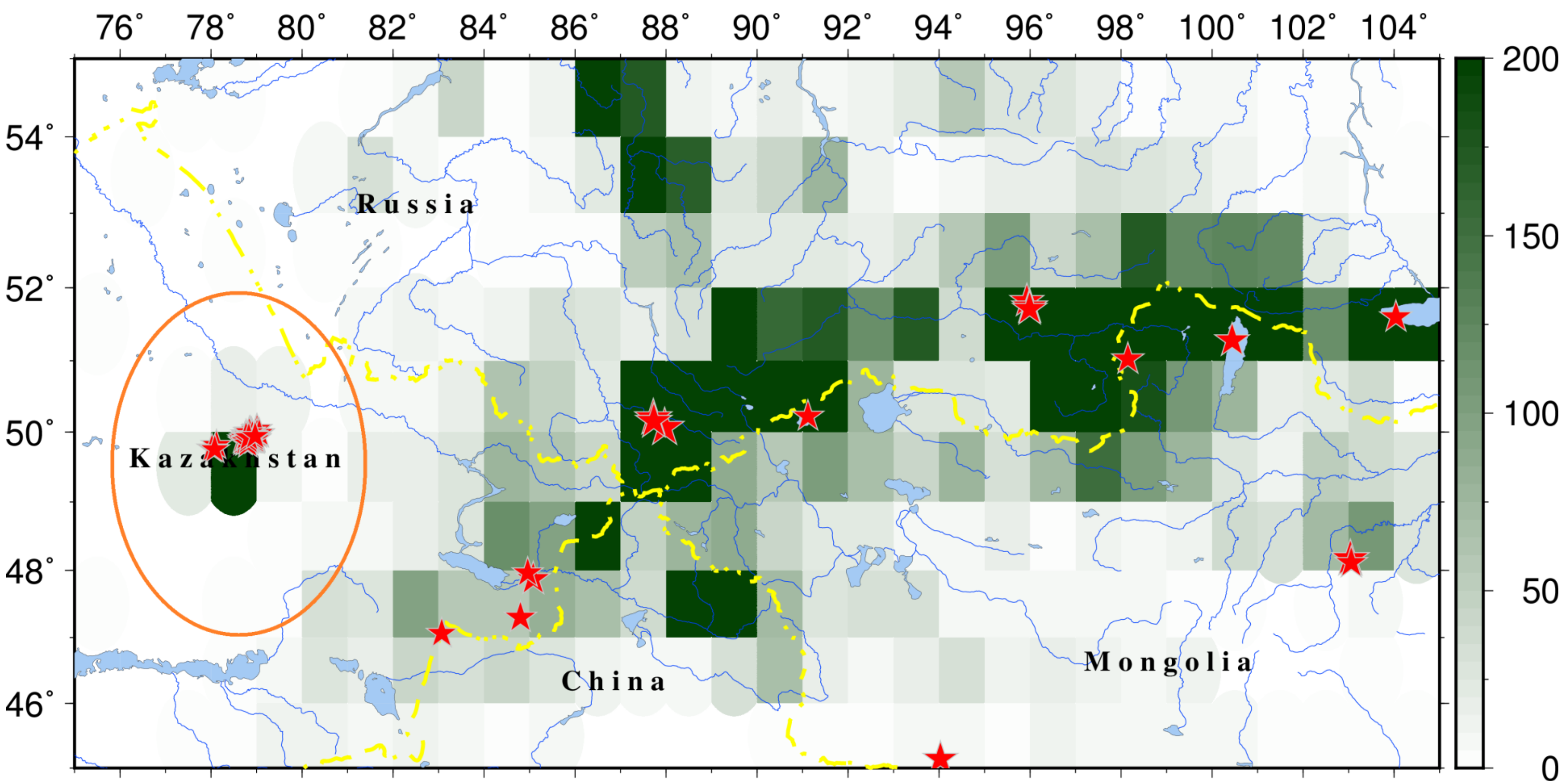
Motivation

Quantitative distribution of earthquakes

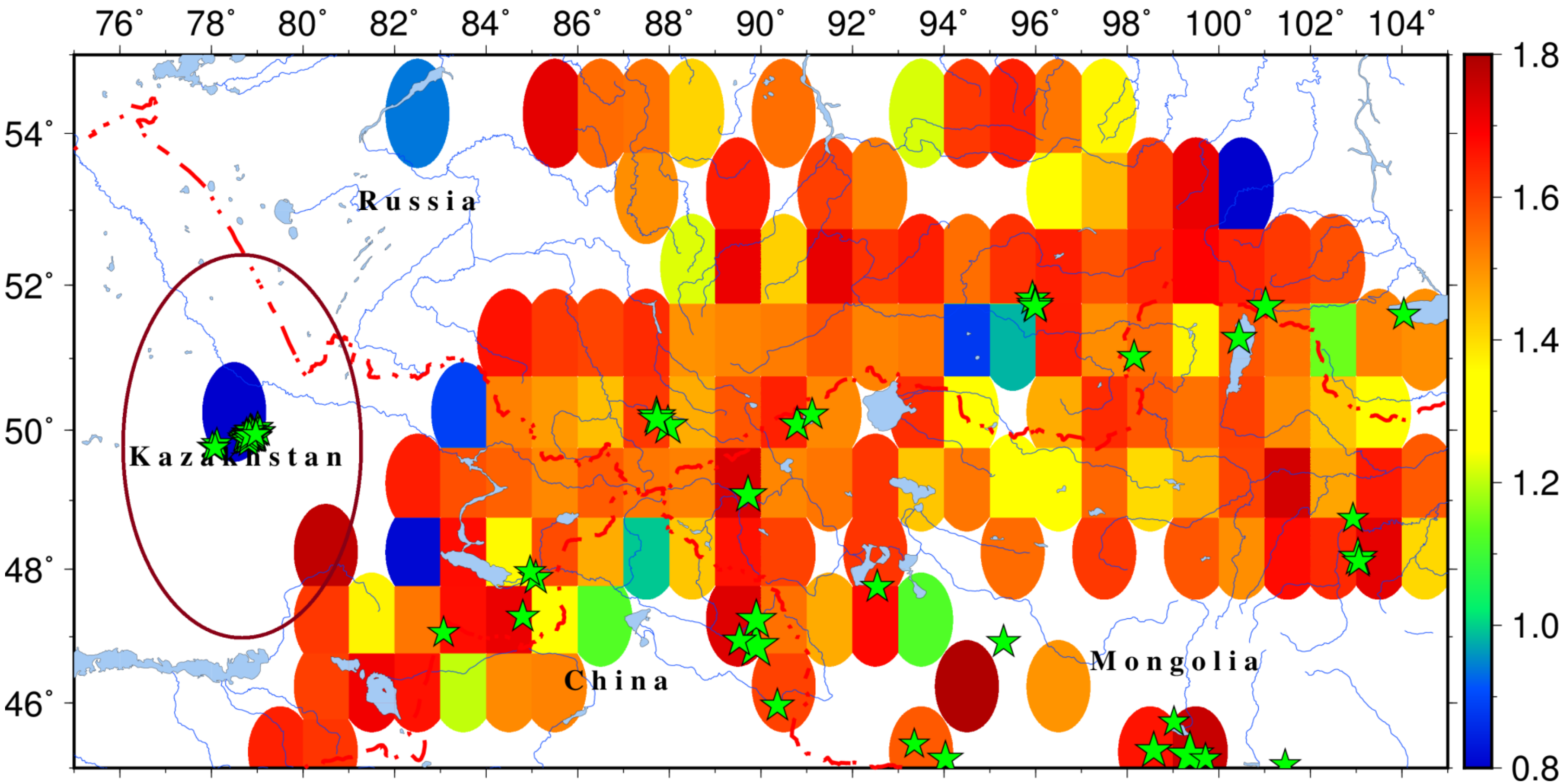


Asterisks - earthquakes with M ≥ 6

Sycheva N.A., Sychev V.N. **2022.** Some characteristics of the seismicity of Altai and Sayan. In: *Materials of the XIV school-conference with international participation "Problems of Geocosmos 2022", Saint Petersburg, October 3–7, 2022*. St. Petersburg: Skifia-print, p. 84–92. (In Russ.).

Motivation

Distribution of the Tsallis parameter *q*



Asterisks - events with M≥6.0.
The area of explosions is characterized by a low level of cross-correlation.

Task

- The purpose of this work is to evaluate the application of an earthquake model based on the principles of non-extensive statistical physics to the seismic process occurring on the island Sakhalin. The task also consists in determining how, within the framework of the model used, the natural seismic process differs from man-made impacts, namely from explosions.

Non-extensive statistical physics (NESP) or q-statistics

In 1988, Constantino Tsallis generalized the classical Boltzmann-Gibbs formula to describe complex non-additive statistical systems, introducing the parameter q into the expression, which characterizes the degree of non-additivity, and proposed the so-called non-extensive or non-additive entropy, which on a discrete number of microstates N is determined by the following expression:

$$S_q = k\frac{1}{q-1}\left(1-\sum_{i=1}^{N} p_i^q\right); \qquad \sum_{i=1}^{N} p_i = 1$$

where p_i is the probability that the system is in the i-state, N is the number of states of the system, k is some positive constant that determines the unit of entropy and in physical formulas serves to connect dimensions, such as the Boltzmann constant.

The Boltzmann statistics corresponds to the $q \to 1$ limit, $q > 1$ indicates the presence of long-range correlations and memory in a non-equilibrium system when additivity is violated.

NESP

Thus, the Tsallis entropy is already a non-extensive function. If, say, the system is divided into two independent subsystems *A* and *B*, then the total entropy of the system will be as follows:

$$S_q(A+B)=S_q(A)+S_q(B)+(1-q)S_q(A)S_q(B)$$

The parameter *q* in this case is a measure of the non-extensiveness of the system under consideration: for $q < 1$, we obtain the over-extensive case, $q = 1$, the extensive case, and, for $q > 1$, the subextensive case.

The approach is used to describe non-equilibrium dissipative systems with memory and long-range correlations, in which the additivity property can be violated, since in such systems each element can interact not only with its nearest neighbors, but also with the entire system as a whole or its large parts.

Tsallis distributions are used to solve problems in the field of statistical mechanics, geology, anatomy, astronomy, economics, finance, and machine learning.

NESP

The probability density distribution function can be described by a *q*-Gaussian

$$G_q(\beta, x) = \frac{\sqrt{\beta}}{C_q} e_q^{-\beta x^2}$$

for 1 <q< 3

$$C_q = \frac{\sqrt{\pi}\,\Gamma\left(\frac{3-q}{2(q-1)}\right)}{\sqrt{(q-1)}\,\Gamma\left(\frac{1}{q-1}\right)}$$

Expression uses expressions of the so-called *q*-algebra, in which the following functions are introduced:

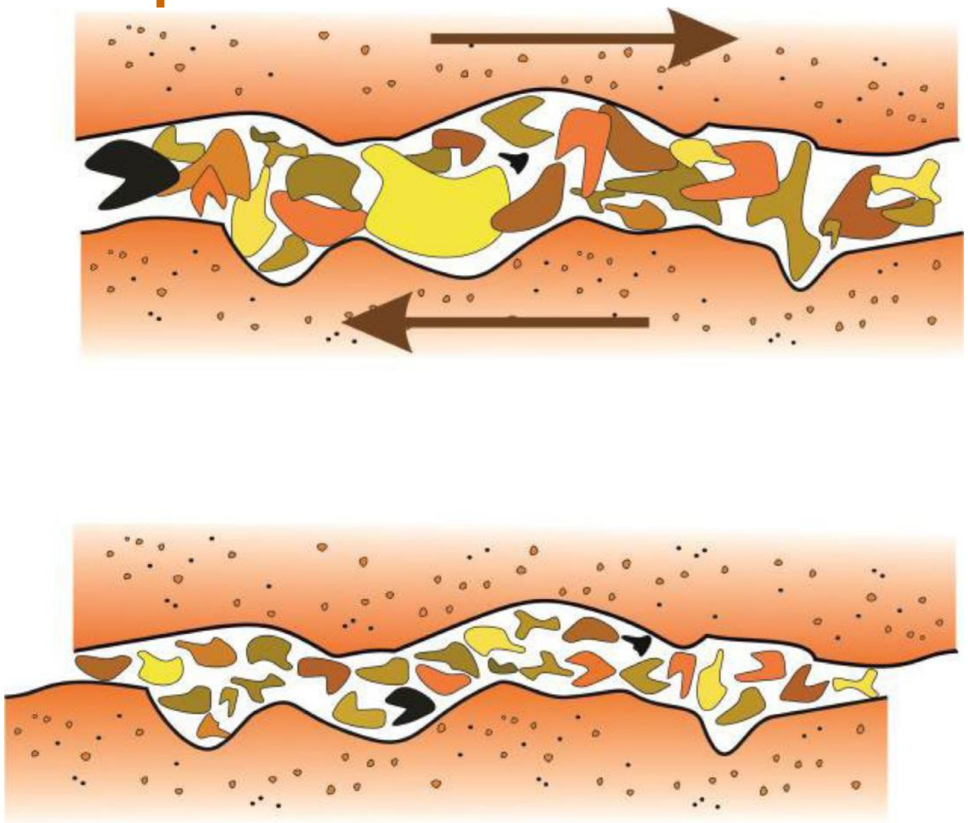
q-exponent

$$e_q^x = [1 + (1-q)x]^{\frac{1}{1-q}},$$

and *q*-logarithm

$$ln_q(x) = \frac{x^{1-q}-1}{1-q}$$

Fragment-Asperity Interaction Model for Earthquakes



Schematic visualization of the fracture contact zone before and after an earthquake.

Posadas A., Sotolongo-Costa O.: Non-extensive entropy and fragment–asperity interaction model for earthquakes. Communications in Nonlinear Science and Numerical Simulation **117**, 106906 (2023)

Fragment-Asperity Interaction Model for Earthquakes

if $N(M > Mth)$ is the cumulative distribution of the number of earthquakes N with a magnitude greater than Mth, then:

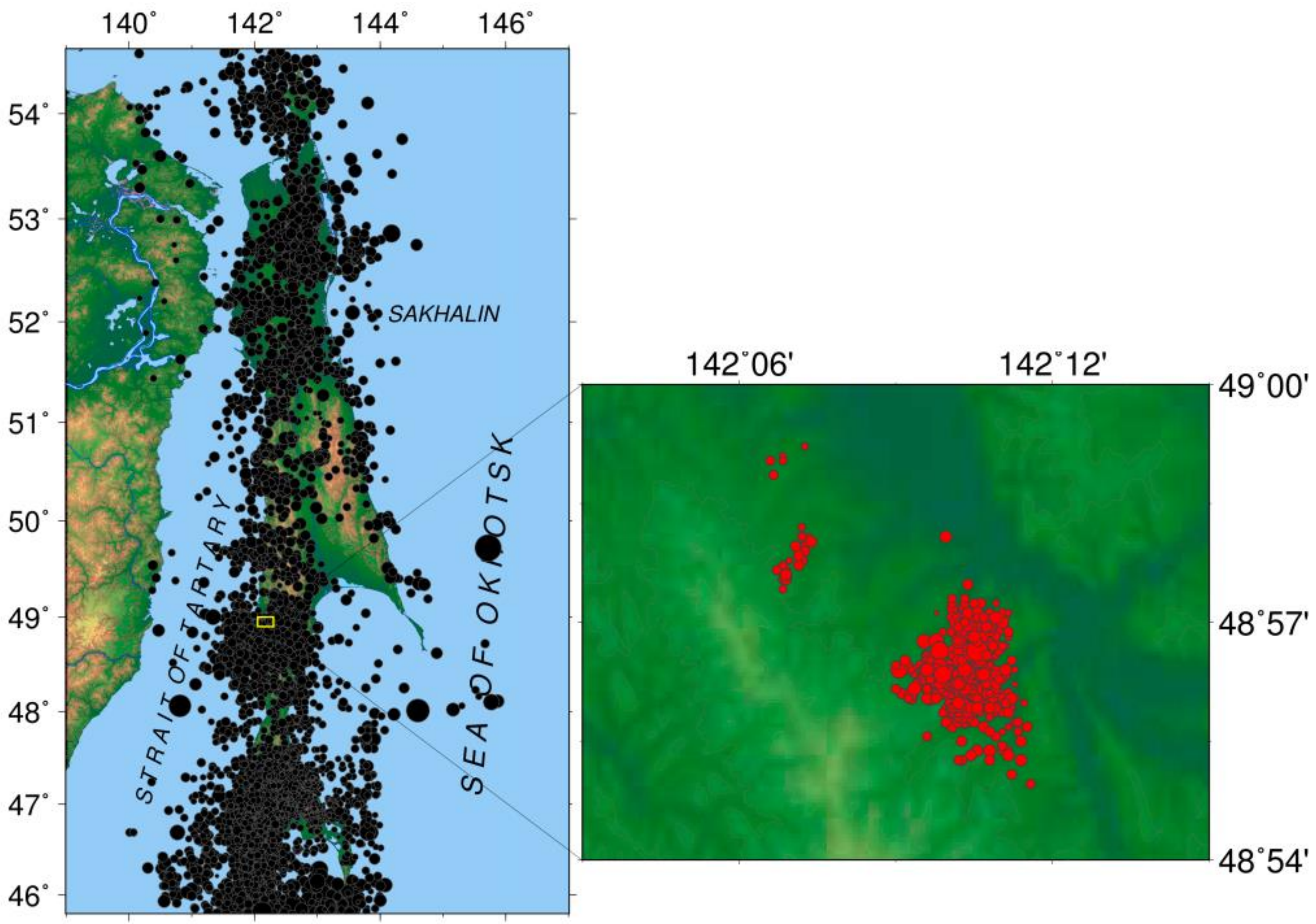
$$\log\left(\frac{N(M>M_{th})}{N}\right) = \left(\frac{2-q}{1-q}\right)\log\left[\frac{1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^{M_{th}}}{a^{2/3}}\right)}{1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^{M_0}}{a^{2/3}}\right)}\right]$$

where M_o is the threshold magnitude in the catalog, a is the coefficient of proportionality between the earthquake energy E and the size of the block fragment r^3 between faults, q is the Tsallis parameter from the expression for the Tsallis entropy. Equation generalizes the Gutenberg-Richter relation in a wide range of values and demonstrates good agreement for various earthquake catalogs.

In fact, the Gutenberg-Richter law can be derived from expression as

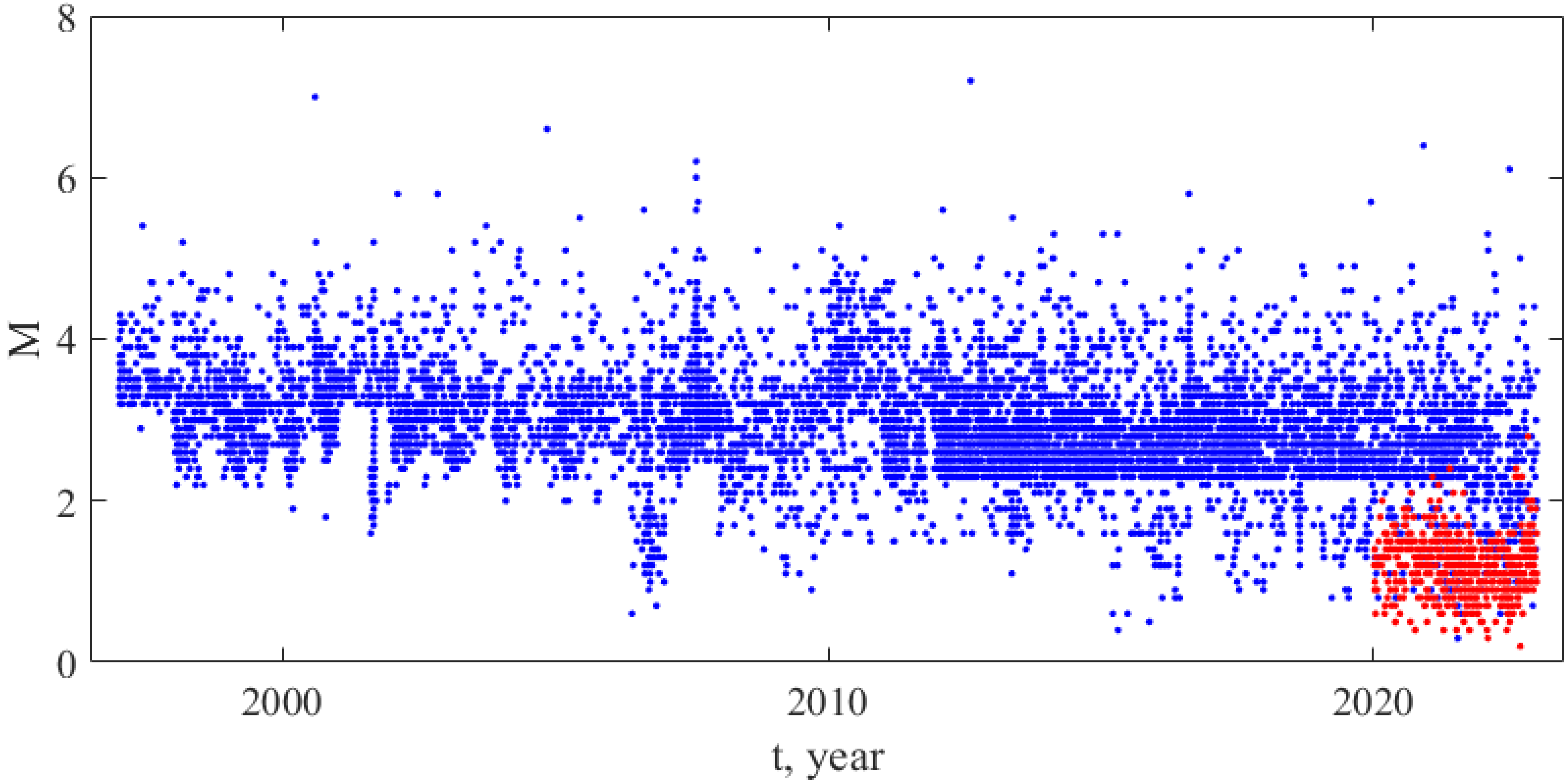
$$b = 2 \cdot \frac{2-q}{q-1}$$

Input data



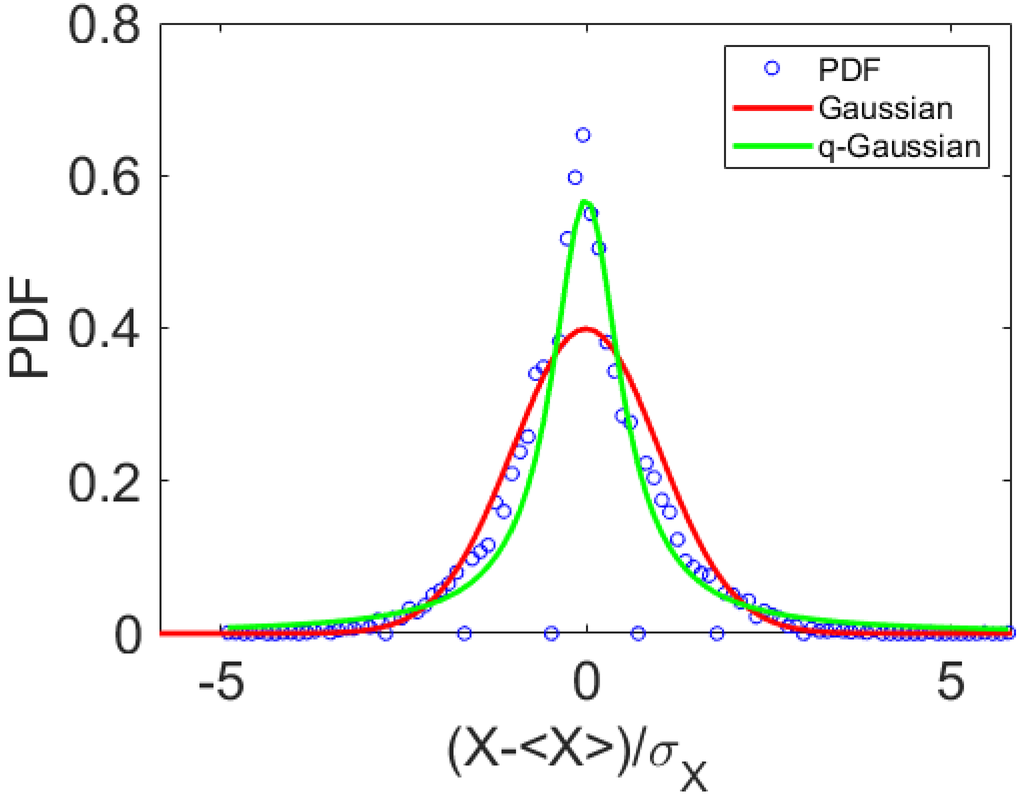
Epicentral position of earthquakes (8127 events) that occurred from 1997 to 2022 The inset shows the explosions (560 events) of the Solntsevsky coal mine for 2020-2022.

Distribution of event magnitudes earthquakes and explosions



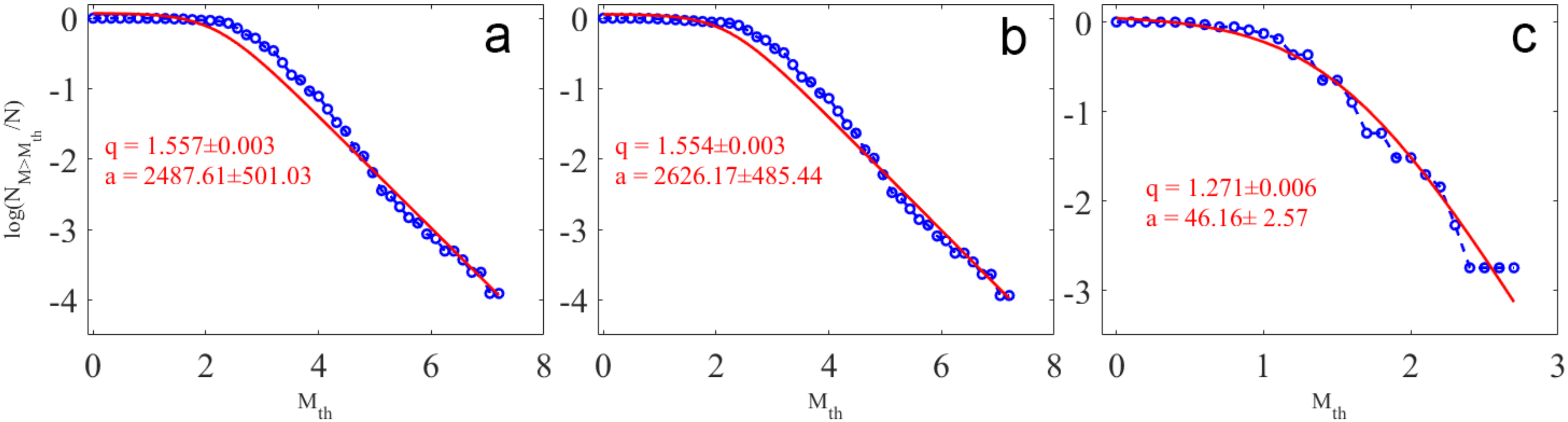
Distribution of event magnitudes from the catalogs of earthquakes and explosions at the Solntsevsky coal mine coal mine over time. Blue dots are earthquakes, red dots are explosions.

Probability density distribution function



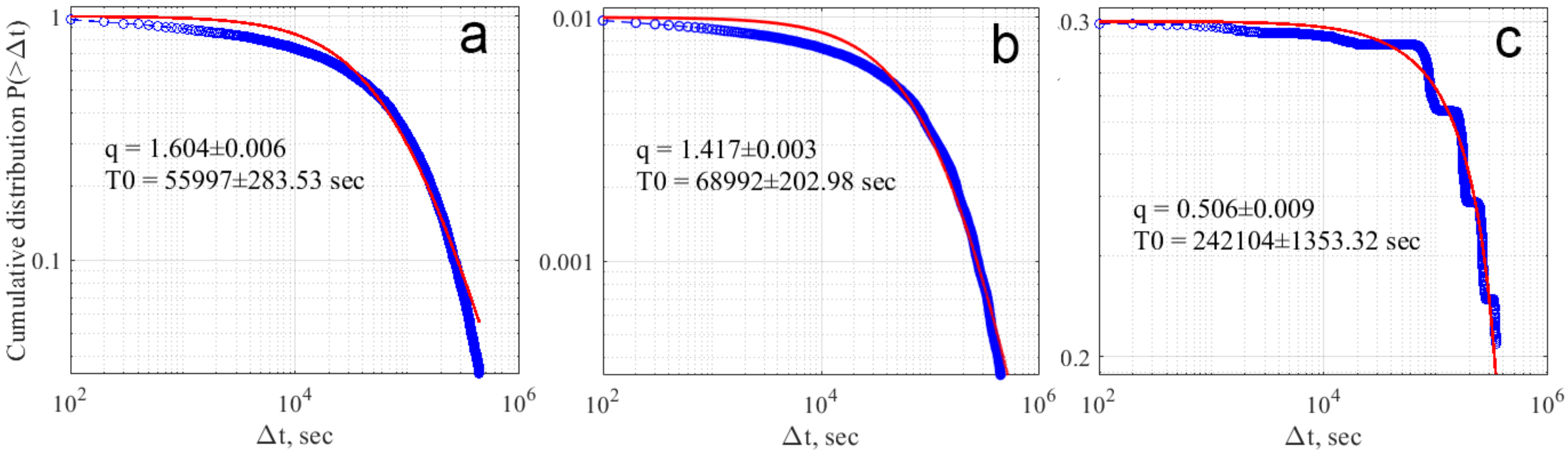
Probability density distribution function of magnitude difference between successive earthquakes and its approximation by Gaussian and *q*-Gaussian with parameters q =2.0119 ±0.03, β =3.1173 ±0.325.

Fragment-Asperity Interaction Model for Earthquakes



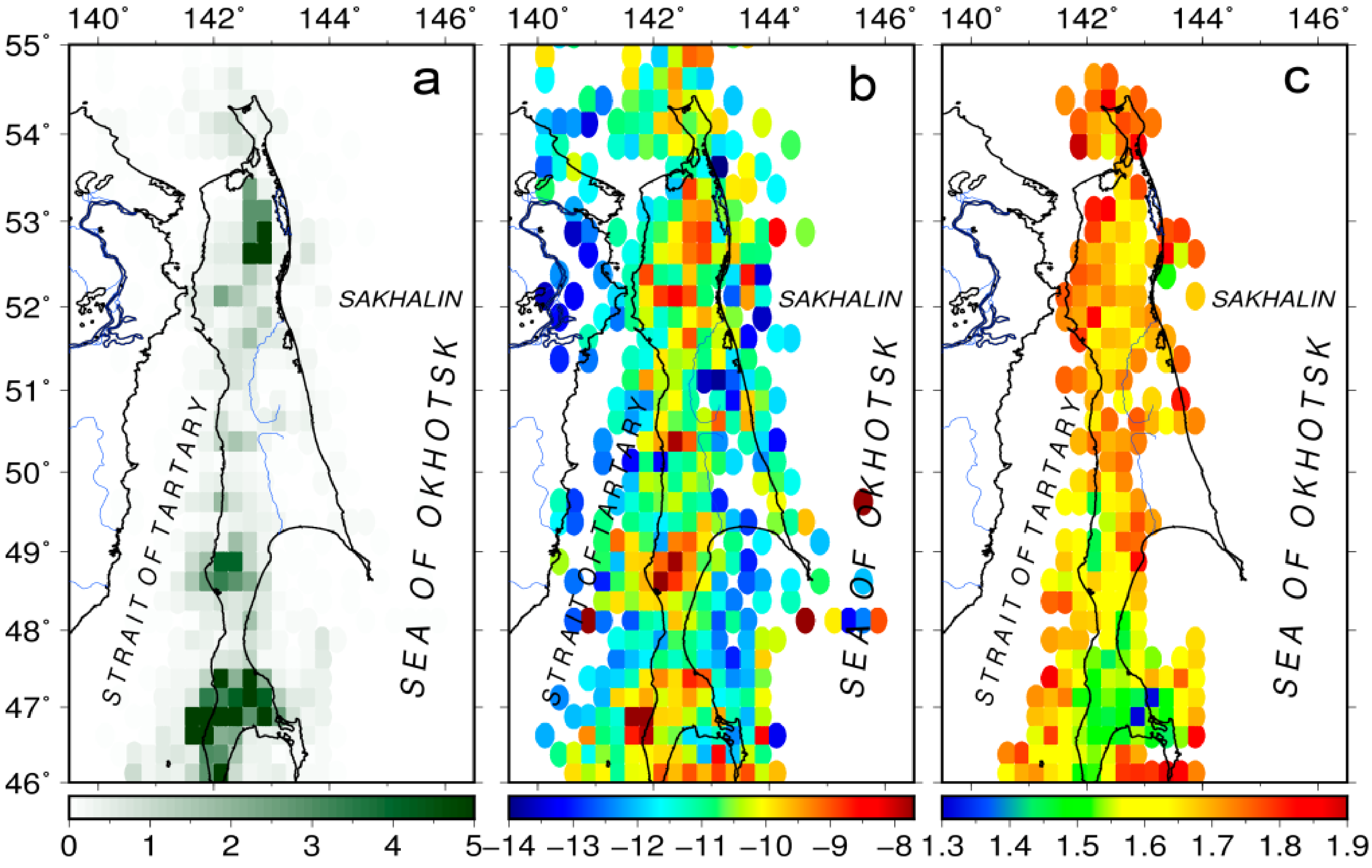
Distribution of the relative number of earthquakes with magnitude $M>M_{th}$ depending on the magnitude: a – earthquake catalog, b – earthquake catalog + explosion catalog, c – explosion catalog.

q-exponent

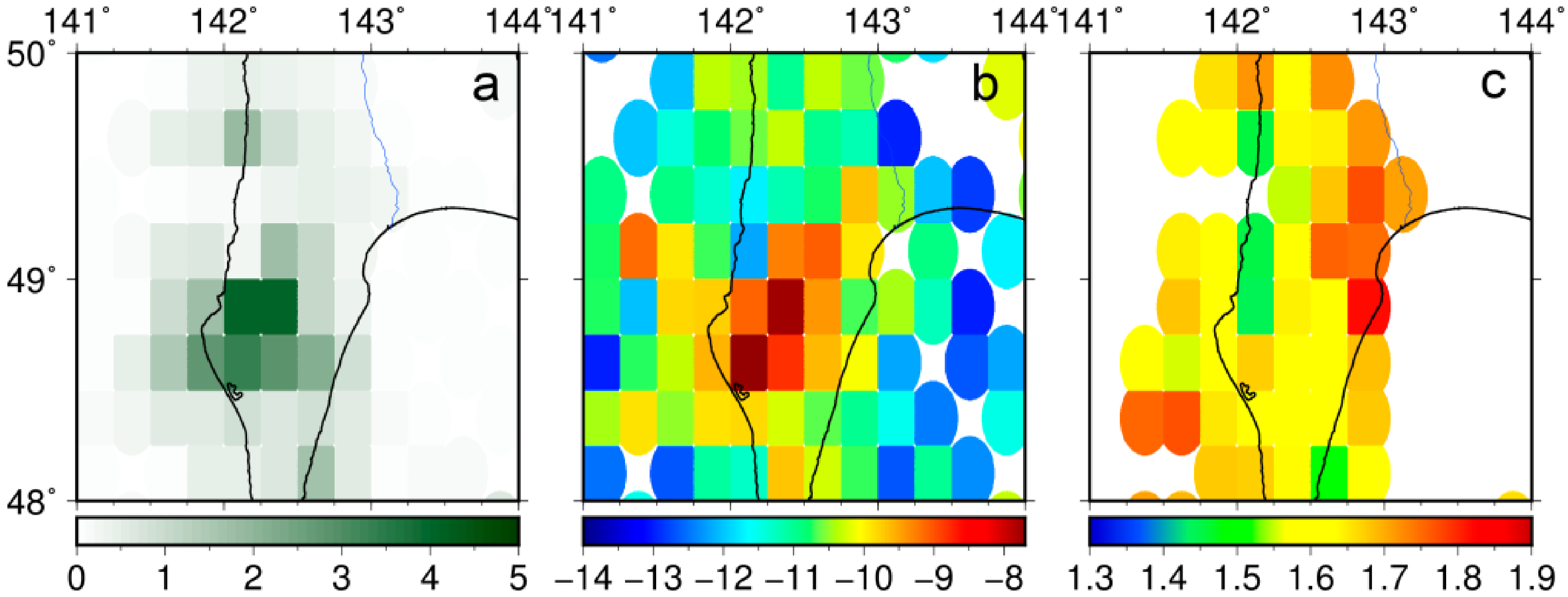


Cumulative distributions of time intervals between successive events: a – earthquake catalog, b – earthquake catalog + explosion catalog, c – explosion catalog

Distribution of the number of earthquakes (a), the logarithm of the STD intensity (b), and the Tsallis parameter *q*(c) over the territory.



Detailed distribution over area: a – number of events, b – STD intensity, c – Tsallis parameter q.



Despite the fact that the number of events and the intensity of STDs demonstrate an active seismic process, the Tsallis parameter *q* has lower values in this region.
Despite the determinism and the presence of periodicities in the catalog of explosions, individual acts of event-explosions themselves weakly or even do not correlate with each other, and even more so with the catalog of earthquakes.

Conclusion

Increased values of the Tsallis q parameter are observed in areas with an active seismic process and coincide with areas of increased seismicity, which correspond to the maps of seismic activity of Sakhalin. Areas where blasting is carried out are marked by lower values of the Tsallis parameter q.

When observing the dynamics of changes in the Tsallis q parameter in mining areas, an increase in this parameter may accompany an increase in the probability of occurrence of underground shocks.