Research of stress-strain state of geo-environment by emanation methods, by example, alpha(t)-model of radon transport

XIII International Conference Solar-Terrestrial Relations And Physics Of Earthquakes Precursors

Tverdyi D.A.[1,2], Makarov E.O.[1,3], Parovik R.I.[1,2]

[1]Kamchatka State University named after Vitus Bering, Russia, Kamchatka State, Petropavlovsk-Kamchatsky, Pogranichnaya str., 4;
[2]Institute of Cosmophysical Research and Radio Wave Propagation FEB RAS, Russia, Kamchatka State, Paratunka, Mirnaya St., 7;
[3] Kamchatka Branch of the Federal State Budgetary Institution of Science Federal Research Center "Unified Geophysical Service RAS", Russia, Kamchatka State, Petropavlovsk-Kamchatsky, Piipa Boulevard, 9

25–29 September, 2023
Paratunka, Russia

Introduction

Continuous monitoring of RVA (radon volumetric activity (^{222}Rn)) variations is one of the techniques for studying the stress-strain state of the geo-environment. [Рудаков, 2009] [Адушкин and Спивак, 2014] [Neri et al., 2011] [Barberio et al., 2018]. Monitoring is carried out in order to search for anomalous RVA values that may be precursors to seismic events [Tsunomori et al., 2011] [Wakita, 1981].

Radon (^{222}Rn) is an inert radioactive gas with half-life $T = 3.85$. It is a daughter decay product of radium (^{226}Ra), which is permanently contained in the Earth's crust. Therefore, ^{222}Rn can be continuously monitored with gas-discharge counters.

Therefore, continuous monitoring of ^{222}Rn emanations in the topsoil in seismically dangerous regions such as Kamchatka [Фирстов and Макаров, 2018], is of interest in terms of developing a methodology for predicting strong earthquakes.

About modelling dynamic ^{222}Rn

As a rule, the instrument for research of the dynamics of ^{222}Rn variations is mathematical modeling. The basis of such models is the application of ODEs or PDEs of integer orders with appropriate initial and boundary conditions [Паровик, 2014].

Many mathematical models have been developed to describe the mechanisms of migration of ^{222}Rn emanation. The emanation method considers migration in groundwater, in porous or fractured geologic media. For example: the hydrothermal system model [Барсуков et al., 1985], the "geogas" model [Varhegyi et al., 1986], physical-chemical model [Понамарев, 1989].

However, within the framework of this research, we are most interested in models based on mechanical concepts. Which take into account the change of vertical velocity of gas flow under the action of tectonic stresses [King, 1991] and radon mixing or injection into groundwater flow [Dubinchuk, 1991].

Existing transfer mechanisms ^{222}Rn

The mechanisms of ^{222}Rn transport in the vertical direction are divided into diffusion and convective [Новиков, 1989]:

- diffusion due to the concentration gradient ^{222}Rn;
- diffusion due to pressure gradient in the Earth's crust;
- thermal-liquid convection due to the lifting force induced by the geothermal gradient;
- gas lifting force in pore medium when pores are filled with water;
- change of pore pressure under the action of changing stresses in the mountain mass;
- turbulent effects in underground air with changing meteorological factors.

Remark 1

At the last stage of earthquake preparation, the structural inhomogeneity of the geosphere lead to the occurrence of «compression–stretching» stress concentration in fracture zones.

On possible causes of the anomalous RVA

The difficulty of searching for earthquake precursors associated with RVA is that in the area of installation of sensors simultaneously can be observed many factors affecting the RVA. The main difficulty is to identify the factor related to changes in the stress-strain state of the medium.

If observations are carried out in an area with a developed hydrochemical system, its overall response to deformation effects is proportional to the integral sum of spatial and temporal variations of the deformation field.

Remark 2

In this case, the internal free energy of molecules of such gases as radon, helium can exceed the threshold of the potential barrier, preventing their exit from the crystal lattice in the interstitial space of the geosphere. As a result, the formation of anomalous concentrations of ^{222}Rn occurs in the subsurface air and in gases dissolved in groundwater.

RVA process data with anomalous variation

The data in (Fig. 1) used in modeling were obtained at the MRZR monitoring station located at the base of the Moroznaya-1 borehole (Elizovsky district) in Kamchatka.

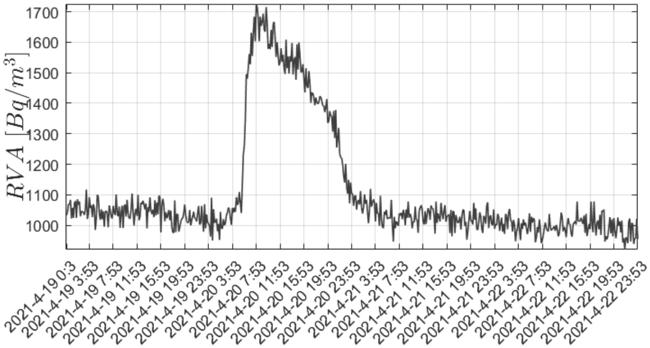

Figure 1: Data on β - radiation accompanying radioactive decay of ^{222}Rn, and characterizing variations of RVA

On recording data on the decay of ^{222}Rn

Remark 3

The location of monitoring points is tied to river basins, as they trace fracture zones of the crust. As the zones of dynamic influence of faults have increased permeability, this will contribute to the flow of subsurface gases into the atmosphere [Рудаков, 2009, Фирстов and Рудаков, 2003].

In particular, data from the MRZR station located at the base of the Moroznaya-1 borehole (Yelizovsky district) were used. The SBM-19 sensor in the accumulation chamber (standard bucket as before) was used. In the point ^{222}Rn is registered in the accumulation chambers at depths of 0.2 and 1.0 meters, in increments of 10 minutes for 96 hours, as shown in (Fig. 1).

Remark 4

Since we are interested in the anomalous variations of RVA, we will use the burst moment from the data presented in (Fig. 1).

Modeling results

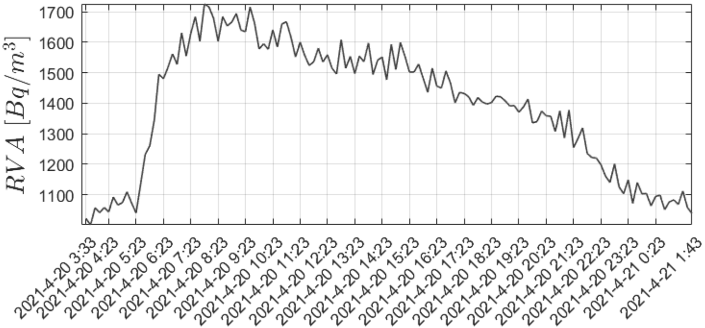

Figure 2: RVA burst extracted from MRZR data, duration 22.5 hours

The surge on (Fig. 2) at the MRZR point is similar in shape to the injection of ^{222}Rn in the water flow due to changes in the stress-strain state of the Earth's crust. In (Fig. 2) we can see a rapid rise, and after some time a smoother decline of RVA. This may be accompanied by a slight increase in RVA values and further decline to the initial level, or close to the initial level.

The well-known classical mathematical model

We will model the RVA process starting from the classical mathematical model based on ODE. For which the Cauchy problem has the form:

$$\frac{dA(t)}{dt} = -\lambda_0 A(t) + S^{\Delta}, \qquad A(0) = A_0, \tag{1}$$

where,

- $A(t) \in C^1[0, T]$ – RVA, Bq$/m^3$;
- S^{Δ} – constant responsible for the diffusion mechanism of transfer, Bq$/(m^3 c)$;
- λ_0 – air exchange rate (AER), c^{-1};
- A_0 – constant determining the value of RVA at the moment of time $t = 0$;
- $t \in [0, T]$ – current modeling time;
- $T > 0$ – total simulation time.

Remark 5

The model(1) we will call it classical due to the fact that it is well studied and often found in multiple works on radon themes [Dubinchuk, 1991] [Фирстов and Макаров, 2018] [Vasilyev and Zhukovsky, 2013].

Preconditions and grounds for model generalization

Remark 6

Porosity of the medium is interpreted as the existence of isolated pores, which slows down the diffusion process (subdiffusion). Permeability of the medium – may be explained by the existence of conductive channels between pores, which leads to acceleration of diffusion (superdiffusion).

These processes belong to the phenomena of anomalous diffusion, which have been studied in detail in [Uchaikin, 2013], and mathematically described using integro-differential calculus [Kilbas et al., 2006, Нахушев, 2003, Псху, 2005]. What connects them with the concept of heredity (or memory) – the characteristic of a system or environment to remember for some time the influence exerted on it.

We proceed from the hypothesis that the ^{222}Rn transfer process takes place in a permeable (porous) geo-environment [Parovik and Shevtsov, 2010]. Therefore, we can generalize the classical model (1) to the case of integro-differential calculus [Kilbas et al., 2006, Нахушев, 2003, Псху, 2005] to account for the memory effect.

Proposed hereditary $\alpha(t)$-model

In such a model, we describe the accumulation process using the nonlinear Riccati equation and the memory effect using the fractional Gerasimov-Caputo derivative of non-integer constant order [Gerasimov, 1948] [Caputo, 1969] :

$$\frac{1}{\theta^{1-\alpha(t)}\Gamma(1-\alpha(t))}\int_0^t \frac{A'(\sigma)}{(t-\sigma)^{\alpha(t)}}d\sigma = -a(t)A(t)^2 - \lambda_0 A(t) + S^{\Delta}(t),$$
$$A'(\sigma) = \frac{dA(\sigma)}{d\sigma}, \qquad A(0) = A_0. \tag{2}$$

where,

- θ – parameter entered to keep the dimensions [Рехвиашвили and Псху, 2022];
- $S^{\Delta}(t) \in C[0, T]$ – time-dependent emanation source function ^{222}Rn, responsible for the diffusive transport mechanism ^{222}Rn, Bq/(m^3c);
- $a(t)$ – coefficient at the component with quadratic nonlinearity, it is supposed that this component is responsible for the flow of ^{222}Rn out of the chamber by atmospheric pressure;
- $\alpha(t)$ – variable order of the fractional derivative. The parameter responsible for the intensity of the transfer process ^{222}Rn, associated with the characteristics of the geo-environment: permeability, porosity [Tverdyi and Parovik, 2022] [Tverdyi et al., 2023].

Remark 7

Note that the ereditary $\alpha(t)$-model RVA (2) when the order of the fractional derivative $\alpha(t) = 1$ and also when $a(t) = 0$ and $S^{\Delta}(t) = S^{\Delta}$ – const, will transform into the classical model RVA (1). This fact indicates the preservation of the properties of the solution obtained earlier by the classical model RVA (1), as well as the existence of new properties applicable to the description of anomalous variations of RVA.

The ereditary $\alpha(t)$-model RVA was solved numerically using a non-local implicit finite-difference scheme (IFDS) of the first order of accuracy. This scheme for the development of a more general Cauchy problem $\partial_{0t}^{\alpha(t)} A(\sigma) = F(A(t), \sigma), \quad A(0) = A_0,$. The numerical scheme is investigated for stability and convergence in [Tverdyi and Parovik, 2022, Tvyordyj, 2021].

About modeling

To compare with the modeling results, the following actions were performed on the RVA variation data in (Fig. 2): smoothing using «Simple moving average» [Johnston et al., 1999] with a window of 2 values; shift to the minimum value; normalization to the maximum value.

The functions $a(t)$, $\alpha(t)$ and the parameter λ_0 were refined from the RVA data so that the solution obtained by the model (2) would give the maximum value of the Pearson correlation coefficient ($Corr$) [Cox and Hinkley, 1979] and coefficient of determination (R^2) [Hughes and Grawoig, 1971, Chicco et al., 2021] with the processed RVA variation data, as presented in (Figure 4).

Example 1

Values of model parameters (2):

$$N = 134, \quad T = 134, \quad A_0 = 0.014, \quad A_{max} = 1, \quad \lambda_0 = 0.05,$$

$$\alpha(t) = 1 - \left(\frac{(T-t)}{T} \cos\left(\frac{3\pi\ t}{T} \right)^2 \right),$$

$$a(t) = -2\lambda_0 + 7\lambda_0 \left(2\cos\left(\frac{\pi\ t}{T} \right)^2 + \cos\left(\frac{2\pi\ t}{T} - \frac{\pi}{11} \right)^2 \right),$$

$$S^{\Delta}(t) = 6\lambda_0 \left(\frac{12(T-t)}{10\ T} \sin\left(\frac{2\pi\ t}{T} \right)^2 + \frac{(T-t)}{T} \sin\left(\frac{3\pi\ t}{T} \right)^2 \right).$$

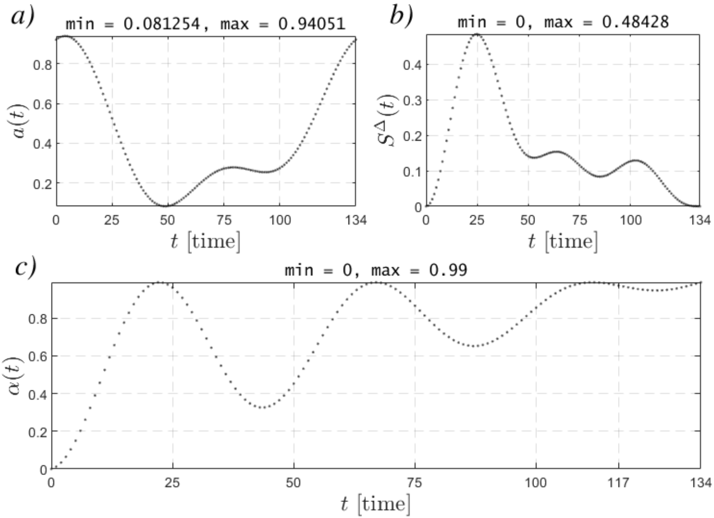

Figure 3: Variation of model parameters as functions of time for Example (1)

Modeling results

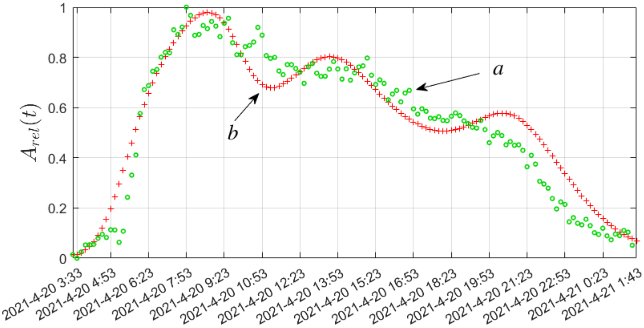

Figure 4: a) Processed RVA burst data from (Fig. 2); b) Modeling results by (2) with coefficient of determination ($R^2 = 0.91$) and Pearson correlation coefficient ($Corr = 0.96$)

In (Fig. 4), a rapid increase in the model values of RVA is first observed. This is due to the intensification of ^{222}Rn transport under the conditions of stress-strain state of the geo- environment. In the model (2) it is represented by the behavior of the parameter $\alpha(t)$, shown in (Fig. 3c). As a result of geo-environment compression, ^{222}Rn is squeezed out of pores and fractures, i.e. its injection, which is represented in the values of $S^{\Delta}(t)$ shown in (Fig. 3b).

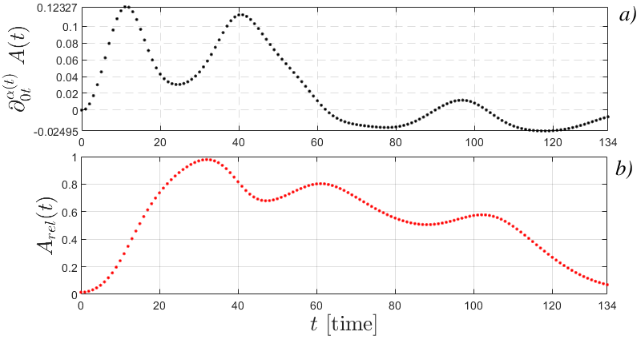

Figure 5: Dependence of: a) fractional derivative values; b) model curve (2) for Example (1)

The memory effect (ereditarity) arising in the model because of the introduction of the fractional derivative $\partial_{0t}^{\alpha(t)} A(t)$ is represented in (Fig.5a). This operator characterizes the intensity of the RVA change and reflects the dissipation of the RVA as shown in (Fig. 5b). Here we some time delay due to the memory effect of this dynamical system.

Interpretation of results

The rapid burst of RVA is probably due to deformations that cause changes in the flux ^{222}Rn through the platform under the storage chamber. The impact of the stress pulse can be of different intensities and durations.

As it was shown above, there are many models describing migration of ^{222}Rn in mountain formations. Based on them, it can be hypothesized that against the background of a constant flux of ^{222}Rn entering the chamber, an overflow of radon may occur. Which is connected with different reaction of the medium to deformation processes (by models from the papers listed earlier).

Excess ^{222}Rn enters the groundwater flow or new fractures formed (depending on the depth or volume of the medium in which it was released). After that it moves by movement in the water flow, diffusion and convection to the ground surface.

This process determines the ascending, relative to background values, part of the anomalous curve of RVA variations.

Since ^{222}Rn decays continuously after its generation, the maximum of the anomalous RVA curve is determined by the volume of released ^{222}Rn and the arrival time from the depth where it started its migration.

Interpretation of results and Conclusion

The descending part (Fig. 4) of the anomalous RVA surge in the storage chamber can be related to the convective air flow through the storage chamber. That is conditioned in the mathematical model (2) by the parameter $a(t)$ and the character of behavior of this function (Fig. 3a).

The air flow through the chamber can be related to, changes in atmospheric pressure at different periods, wind blasts, and warming of the upper ground layer[Фирстов et al., 2018].

Since the described anomalous RVA process takes place in less than 1 day, radon decomposition can be ignored when describing the descending part of the RVA in (Fig. 4). In the case of stopping its excess flow into the chamber.

Remark 8

Therefore, the processes that lead to the appearance of excessive volume of radon in the storage chamber can be related to:

- *stretching of the medium (increase of free pores, formation of new fractures, filling of pores with fluid and pushing of gas radon to the surface);*
- *compression of the medium (radon extrusion, increase of emanation due to excess energy due to heating of the medium from friction, desorption due to microvibrations, etc.);*
- *as well as with changes in convective flow of subsurface air.*

Conclusion

- An ereditary $\alpha(t)$ RVA model was developed and applied to describe the dynamics of the accumulation of ^{222}Rn taking into account the memory effect.
- The model has been tested at one monitoring point ^{222}Rn of the Petropavlovsk-Kamchatsky geodynamic polygon.
- It is shown that the proposed mathematical model well describes more complex impulses (bursts) of RVA due to the specific type of functions entering the model equation.
- An interpretation of the modeling results is given.
- It is shown that the order of the fractional derivative may be responsible for the intensity of the ^{222}Rn transfer process (memory effect), which is related to the characteristics of the geo-environment: porosity, permeability, etc.
- It is shown that the nonlinear component in the model equations determines the law of accumulation of ^{222}Rn close to the logistic one and is described by the Riccati equation. Such nonlinearity gives fast increasing of RVA values and reaching saturation – some constant level.

Финансирование/Funding

Исследования выполнены в рамках гранта Президента РФ МД-758.2022.1.1 по теме "Развитие математических моделей дробной динамики с целью исследования колебательных процессов и процессов с насыщением"

The work was carried out within the framework of the grant of the President of the Russian Federation MD-758.2022.1.1 on the topic "Development of mathematical models of fractional dynamics in order to study oscillatory processes and processes with saturation"

Thanks for your attention!

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